Math 1242 Key

Review

1. Assume the functions described below have both a first derivative and a second derivative everywhere. Answer each of the following using the appropriate response from
POSITIVE, NEGATIVE, ZERO, or CANNOT DETERMINE

a. If f is increasing at x=3, then

f'(3) = positive   f''(3) = cannot be determined

b. If f has a relative maximum at x=7, then

f'(7) = zero   f''(7) = Negative

CD

c. If f has a relative minimum at x=-6, then

f'(-6) = zero   f''(-6) = Positive

CU

d. If f is decreasing at x=32, then

f'(32) = negative   f''(32) = cannot be determined

e. If f has an inflection point at x=41, then

f'(41) = Cannot Determine   f''(41) = Zero

2. Assume that f is differentiable everywhere and

$f'(0) = 8/9$	$f''(0) = -2$	$f'(2) = 1/4$
$f'(3) = 0$	$f''(3) = -1$	$f'(5) = -3$
$f''(5) = 1$	$f'(7) = 0$	$f''(7) = 5/3$

a. List two points where f is increasing.

$x = 0$ OR $(0, f(0))$

$x = 2$ OR $(2, f(2))$

b. Where does f have a relative maximum?

$x = 3$ OR $(3, f(3))$

c. Where does f have a relative minimum?

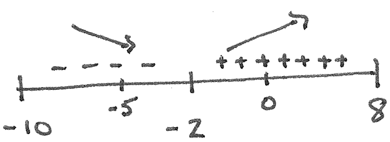
$x = 7$ OR $(7, f(7))$

3. Consider the function $f(x) = 3x^2 + 12x - 36$ on $[-10, 8]$

a. Find where f is increasing and where f is decreasing.

$f'(x) = 6x + 12 = 0$

$x + 2 = 0$



increasing : $(-2, 8)$ OR $x > -2$

decreasing : $(-10, -2)$ OR $x < -2$

b. Find where f is concave upward and where f is concave downward.

$f''(x) = 6$

concave upwards everywhere

ie: CU on $(-10, 8)$

CD on $\emptyset$

c. List all candidates for relative maximums and relative minimums.

$x = -10$
$x = 8$ $\}$ endpoints

$x = -2$ $\}$ since $f'(-2) = 0$

d. Determine:

The relative maximums $x = 8$ and $x = -10$

The relative minimums $x = -2$

$f(x) = 3x^2 + 12x - 36$

$f'(x) = 6x + 12$

e. Find the line tangent to f at x=1.

$m = f'(1) = 6 + 12 = 18$

$f(1) = 3 + 12 - 36 = 3 - 24 = -21$

$y - y_1 = m(x - x_1)$

$y - (-21) = 18(x - 1)$

$y = 18x - 39 \leftarrow$ tangent line at $x = 1$

4. Differentiate each of the following functions (ie: find the derivatives).

a. $f(x) = x^5 - 3x^2 + 11$

$f'(x) = 5x^4 - 6x$

b. $f(x) = (5x^2 + 7x + 6)^7$

$f'(x) = 7(5x^2 + 7x + 6)^6 (10x + 7)$

c. $f(x) = \frac{7}{x^3} = 7x^{-3}$

$f'(x) = -21x^{-4} = \frac{-21}{x^4}$

d. $f(x) = x^3(5x^2 + 7x + 6)^7$

$$f'(x) = 3x^2(5x^2 + 7x + 6)^7 + x^3 \cdot 7(5x^2 + 7x + 6)^6(10x + 7)$$

$$= 3x^2(5x^2 + 7x + 6)^7 + 7x^3(10x + 7)(5x^2 + 7x + 6)^6$$

$$= x^2(85x^2 + 70x + 18)(5x^2 + 7x + 6)^6$$

~~e. f(x) = $\frac{7}{x^2}$~~

f. $f(x) = x^3\sin(x)$

$$f'(x) = 3x^2\sin x + x^3\cos x$$

g. $f(x) = \sin(x^3)$

$$f'(x) = 3x^2\cos(x^3)$$

h. $f(x) = \tan(x)\sec(x)$

$$f'(x) = (\sec^2 x)\sec x + \tan x(\sec x\tan x)$$
$$= \sec^3 x + \sec x\tan^2 x \underset{\text{OR}}{=} 2\sec^3 x - \sec x$$

i. $f(x) = \frac{(x^2+x)^4}{3x+1}$

$$f'(x) = \frac{4(x^2+x)^3(2x+1)(3x+1) - (x^2+x)^4(3)}{(3x+1)^2} = \frac{(21x^2+17x+4)(x^2+x)^3}{(3x+1)^2}$$

j. $f(x) = \frac{6x^2-2x+7}{5x^2+4x+7}$

$$f'(x) = \frac{(12x-2)(5x^2+4x+7) - (6x^2-2x+7)(10x+4)}{(5x^2+4x+7)^2}$$

$$= \frac{34x^2+14x-42}{(5x^2+4x+7)^2}$$

k. $f(x) = \left(\frac{x^2+1}{3x+7}\right)^3 \sin(4x)$

$$f'(x) = 4\left(\frac{x^2+1}{3x+7}\right)^3 \cdot \cos(4x) + 3\left(\frac{x^2+1}{3x+7}\right)^2 \sin 4x\left(\frac{2x(3x+7) - 3(x^2+1)}{(3x+7)^2}\right)$$

$$= 4\left(\frac{x^2+1}{3x+7}\right)^3\cos(4x) + \frac{3\sin 4x\,(x^2+1)^2(3x^2+14x-3)}{(3x+7)^4}$$